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UNRESOLVED RESONANCE PARAMETERS OBTAINED FROM POLARIZATION MEASUREMENTS ON ^{235}U

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ABSTRACT

Recent measurements using polarized neutrons and a polarized ^{235}U target are analyzed with the objective of providing guidance to evaluation efforts for ENDF/B-V. This study is particularly addressed to the unresolved resonance region and above, where fluctuations are observed in the partial cross sections. We find strong evidence to support the hypothesis that these fluctuations are associated with local enhancements due to the double-humped fission barrier. We discuss the applicability of these data in improving estimates for various average parameters (level density, fission width, radiative capture width, s- and p-wave strength functions) and arrive at a recommended procedure for evaluating the observed structure.

INTRODUCTION

The existence of pronounced structure in the neutron-induced fission and total cross sections of ^{235}U below $\sim$ 100 keV is well established, and several analyses have been performed [1-4] which indicate that the structure in the fission cross section cannot be explained by the usual statistical model treatment of unresolved resonances. It has been suggested [1,3,4] that the fluctuations can be attributed to modulations or local enhancements due to states in the second well of the double-humped fission barrier. If this suggestion is correct, it would imply that the present treatment of unresolved resonance cross sections using evaluated data from ENDF/B is inadequate, and could lead to substantive differences in the calculation of self-shielding factors, reactivity coefficients, and the general treatment of cross sections for reactor design.

The only mechanism which is known to lead to intermediate structure in fission is enhancement of the fission widths by states of the second well of a double-humped fission barrier (Class II states). Cao [1] has pointed

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out that the observed frequency of the fluctuations in ($^{235}U + n$) is consistent with the systematics of sub-threshold fission for non-fissile targets and of second well parameters deduced from fission isomers. This mechanism requires that the fluctuations be produced by Class II states of definite spin. This has been experimentally verified by Keyworth et al [5] for ($^{237}Np + n$). Thus we expect that if the structure in ($^{235}U + n$) arises from such a mechanism, the statistical tests which indicate non-statistical behavior in the fission cross section should show this spin dependence.

The technique of using polarized neutrons on a polarized target of ^{235}U, as the definitive method of determining the spins of resonances in the compound nucleus ^{236}U has been discussed by Keyworth et al [6,7], who reported spin assignments to 60 eV. In 1974, a second series of runs was made by Keyworth et al on the Oak Ridge Electron Linear Accelerator with increased polarization. A preliminary report of the results obtained was given at the 1974 Nuclear Cross Sections and Technology Conference [8]. These data extend from 1 eV to 50 keV, and contain high enough statistical accuracy to permit a more nearly complete analysis to be carried out over the entire resonance region, both resolved and unresolved. It should be pointed out that the polarized-neutron-polarized-target technique gives definitive results only for s-wave neutron resonances, which implies that the range of applicability roughly corresponds to the current ENDF/B definition of the resonance region for ^{235}U: 0 - 25 keV.

SUMMARY OF THE EXPERIMENTAL MEASUREMENT AND DATA REDUCTIONS

A complete description of the polarization measurement is not necessary to the present discussion, but a brief summary is provided to show the unique properties of the results obtained. The neutron beam was polarized by transmission through $La_2\,Mg_3\,(NO_3)_{12} \cdot 24\,H_2O$ in which the hydrogen in the water of hydration was polarized. The target was a polarized sample of ^{235}US. The data consisted of time-of-flight spectra of fission events occurring in the sample with the neutron beam polarized parallel and antiparallel to the target, and of the transmitted neutron beam under the same conditions.

For present purposes it is adequate to represent the spin 3^- and spin 4^- enhanced count rates by

$$N_3 = A_3\sigma_3\phi + A_4\sigma_4\phi, \quad (1a)$$

and

$$N_4 = B_3\sigma_3\phi + B_4\sigma_4\phi, \quad (1b)$$

where σ_3 and σ_4 are the spin-3 and spin-4 cross sections, ϕ is the flux, and the constants A_3, A_4, B_3, B_4 are calculated from known neutron polarizations, the nuclear polarization, and the target spin. Equations (1a) and (1b) are solved for the quantities

$$\sigma_4\phi = (A_3N_4 - B_3N_3)/(A_3B_4 - B_3A_4) \quad (2a)$$

and

$$\sigma_3\phi = (B_4N_3 - A_4N_4)/(A_3B_4 - B_3A_4) \quad (2b)$$

These quantities are plotted for energy regions 8 - 20 eV (in the resolved range) and 200 - 260 eV (in the unresolved range), in Figs. 1 and 2. From such plots, it is easy to make spin assignments for essentially all the observed resonance structure and to extract average or effective J values for broad bins in the unresolved region. Here we define $J_{effective} = 3 + \sigma_4/(\sigma_3 + \sigma_4)$. It should also be noted that these data show clearly the existence of previously unresolved doublets of different spin -- for example, the weak spin-3 resonance at 9 eV.

SPIN DEPENDENCE OF STRUCTURE IN THE UNRESOLVED RESONANCE REGION

The question to be addressed is whether the large fluctuations in the fission cross section are spin dependent. In the summed counts ($N_3 + N_4$), the fluctuations are clearly seen, as shown in Fig. 3 for the energy range 8 - 20 keV. However, visual inspection of the spin-separated data, shown in Fig. 4 over the same energy region, shows only slight evidence that any of this structure is associated with one spin or the other. The statistical accuracy of the data is low, and we might assume that it requires quantitative statistical tests on broad-bin averages to reveal any spin dependence. Following Migneco et al [4], we first carried out a Wald-Wolfowitz runs-distribution test from 0.1 to 25 keV on $J_{eff} - \langle J_{eff} \rangle$ with bins of 240 and 400 eV, and from 0.1 to 10 keV with bins of 85 eV. Migneco et al reported that this test gave highly significant results when applied to σ_f for ^{235}U, but the test applied to the polarization data gave results consistent with a random distribution of spin. We next calculated the serial correlation coefficients of J_{eff} with a bin size of 240 eV from 0.1 to 25 keV, followed by a Wald-Wolfowitz test on these coefficients; the same test was also used by Migneco et al [4]. The results again showed no significant departure from a random distribution. Following James et al [3], we tried the Levene-Wolfowitz runs-up-and-down test on J_{eff} with a bin width of 240 eV from 0.1 to 25 keV. Again, the results were completely consistent with the null hypothesis of a random distribution.

The next test, however, showed a much more interesting result. We calculated the correlation coefficient between the spin-3 data and the summed counts and between the spin-4 data and the summed counts, for broad-bin averages. The results, shown in Table I, indicate that the observed structure is attributable to spin 4. Apparently there is still enough statistical error associated with the broad-bin averages that it masked the effect when we used the usual tests for intermediate structure. The results shown in Table I, however, are definitive, showing that essentially all the fluctuating part of the ^{235}U fission cross section has J = 4.

We conclude that the polarization data give strong support to the hypothesis that the fluctuations in the fission cross section of ^{235}U are a second-well phenomenon. We note that the general procedure used in previous versions of ENDF/B for the unresolved resonance region should be modified in order to treat this phenomenon properly.

AVERAGE PARAMETERS FOR THE UNRESOLVED RESONANCE REGION

The polarization data can also be used to provide better estimates of the average parameters for the unresolved region. The first of these is the level density. Fig. 5 shows the usual stairstep distribution of spacings for spin-3 and spin-4 resonances below 360 eV. (Only the tips of the stairs are plotted.) Below 60 eV, we used the Δ_3 test of Dyson and Mehta [9] as a criterion for arriving at the recommended average spacing of 1.153 eV and 0.896 eV for spin 3 and spin 4, respectively, which would imply a total of 119 levels of both spins between 0 and 60 eV. If the spacing distribution follows the prediction of the Gaussian Orthogonal Ensemble (GOE), then the Δ_3 test as a missing-level indicator is a very powerful one. Jain and Blons [10] have questioned the applicability of the GOE for nuclides near A = 240. To check this for (^{235}U + n), we have devised an independent missing level estimator, which is based on two assumptions: (1) the neutron width distribution is Porter-Thomas; and (2) the larger widths are accurately known. For the resonance region in (^{235}U + n), a lower limit of $\langle \Gamma n^\circ \rangle /4$ seems appropriate. It can be easily shown that the Porter-Thomas distribution has the following properties:

$$\int_{1/4}^{\infty} f(x)dx = 0.617, \tag{3a}$$

$$\int_{1/4}^{\infty} \sqrt{\Gamma n^\circ} f(x)dx = 0.704 \langle \Gamma n^\circ \rangle^{1/2}, \tag{3b}$$

and

$$\int_{1/4}^{\infty} \Gamma n^\circ f(x)dx = 0.969 \langle \Gamma n^\circ \rangle , \tag{3c}$$

where $x = \Gamma n^\circ / \langle \Gamma n^\circ \rangle$, and $f(x) = \frac{1}{\sqrt{2\pi x}} \exp(-x/2)$.

If one forms the ratio:

$$\sum_{\langle \Gamma n^\circ \rangle /4}^{\infty} g\Gamma n^\circ \Big/ \Big(\sum_{\langle \Gamma n^\circ \rangle /4}^{\infty} \sqrt{g\Gamma n^\circ} \Big)^2,$$

it has the expectation value

$$\frac{0.969}{(0.704)^2} \cdot \frac{0.617}{n} = 1.206/n$$

where n is the number of levels having Γn° larger than $\langle \Gamma n^\circ \rangle /4$. To use the missing-level estimator, one calculates the quantity $n \, \Sigma g\Gamma n^\circ / (\Sigma \sqrt{g\Gamma n^\circ})^2$, starting with the largest value of $g\Gamma n^\circ$ in the interval and adding additional levels, one at a time, going from larger to smaller values in the ordered array of observed values of $g\Gamma n^\circ$. When this quantity equals 1.206, the total number of levels in the interval is n/0.617. It should be noted that the estimator is independent of any assumptions of $\langle g\Gamma n^{\circ}\rangle$; in fact, an estimate of this quantity is derived along with the total number of levels. We tested the missing-level estimator by Monte-Carlo sampling from a Porter-Thomas distribution as shown in Fig. 6; the expected relative error varies as $1/\sqrt{N}$, where N is the total number of levels in the sample, or $\sim$ 9% for 120 levels.

To use the missing-level estimator for $(^{235}U + n)$, we first note that the s-wave neutron strength function, $\langle\Gamma n^{\circ}/D\rangle$, as calculated from Mughabghab's recommended parameters [11], is independent of spin. The spin independence of the s-wave strength function and the almost perfect agreement of the stairstep spacing distribution (Fig. 5) with the expected (2J + 1) slope below 60 eV suggests that we can use the quantity $g\Gamma n^{\circ}$ as a spin-independent variable in checking for missing levels. It may be useful to point out that the strength function is protected against missing levels so that the spin independence of the strength function is valid even if we miss more levels of one spin than of the other.

We used this estimator with three recommended sets of resonance parameters for ^{235}U, those of Mughabghab [11], those of Smith and Young [12] for ENDF/B-III, and those of Reynolds [13] for ENDF/B-V; the estimator gives 107 ± 10, 117 ± 10, and 110 ± 10, respectively, as the total number of levels of both spins between 0 and 60 eV. These results are consistent with the 119 predicted by using the spacings obtained with the Δ_3 test. We conclude that the GOE gives an accurate representation of the spacing distribution, and that the (2J + 1) variation of the level density seems to be confirmed for ^{235}U; we see no need for a spin cutoff factor, at least for spins $\leqslant 4$. The number of levels which are missed in the usual type of measurement (in which the spins are not separated) seems to be substantially lower than the statistical analysis of Garrison [14] would indicate. We see no evidence for a large number of missing levels as suggested, for example, by Felvinci et al [15].

The average fission widths for the two spin states are different -- the three sets of recommended parameters [11-13] suggest that $\langle\Gamma_f\rangle_{3-}$ is about twice as large as $\langle\Gamma_f\rangle_{4-}$. The resolved resonance parameters of Smith and Young [12] and of Reynolds [13] are based on multilevel analysis of total and all measured partial cross sections, and should be a more accurate representation than those of Mughabghab [11]. The results of the two multilevel fits do not agree, however. Using the Smith and Young parameters, we get $\langle\Gamma_f\rangle_{3-}$ = 0.179 eV, $\langle\Gamma_f\rangle_{4-}$ = 0.090 eV; using the Reynolds parameters we get $\langle\Gamma_f\rangle_{3-}$ = 0.220 eV, $\langle\Gamma_f\rangle_{4-}$ = 0.098 eV. The discrepancy can be attributed to the assumed value for the radiation width: Smith and Young obtain $\langle\Gamma_\gamma\rangle$ = 0.0355 eV; Reynolds uses 0.042 eV. The ratios $\langle\Gamma_f\rangle/\langle\Gamma_\gamma\rangle$ agree; we obtain $\langle\Gamma_f\rangle_3/\langle\Gamma_\gamma\rangle$ = 5.18 and $\langle\Gamma_f\rangle_4/\langle\Gamma_\gamma\rangle$ = 2.45 for the energy range 0 - 60 eV, using the average of both multilevel analyses [12,13]. We prefer the narrower set of widths from Smith and Young [12] for two reasons. First, we expect that narrower widths will give better agreement with the resonance self-shielding experiments of Bramblett and Czirr [16-18], and secondly, we find that an average capture width of 0.042 eV appears to be less consistent with nuclear systematics than is 0.0355 eV. We can calculate the energy dependence of the average radiation width [19], which can be normalized at the neutron binding energy (less the pairing correction) to data for non-fissile targets in the lower actinides. The pairing correction we obtain from a plot of the reduced level spacings D(2J + 1), which also shows a systematic excitation energy dependence, as may be seen in Fig. 7 [20].

The results of this exercise are shown in Fig. 8; they suggest a value of $\langle \Gamma_\gamma \rangle = 0.037$ eV for ^{235}U, although the scatter of data points does not preclude any value in the range of 0.035 to 0.040. We see little reason to change the value of $\langle \Gamma_\gamma \rangle = 0.035$ eV recommended by Pitterle et al [21] for ENDF/B-III.

Using the Smith and Young average radiation width of 0.0355 eV gives $\langle \Gamma_f \rangle_3 = 0.184$ eV and $\langle \Gamma_f \rangle_4 = 0.087$ eV for the resolved resonance region. It is instructive to see what the Bohr-Wheeler estimate would be. Using a single-humped barrier, the estimate is

$$\left\langle \frac{\Gamma_f}{D} \right\rangle = \frac{n}{2\pi} \tag{4}$$

where n is the number of open fission channels. If the barrier has more than one hump, and if the compound nucleus assumption is valid for states in the second well, then the reaction rate follows the expression given by Eyring [22] for sequential processes:

$$k' = \left(\sum_i k_i^{-1}\right)^{-1} \tag{5}$$

where k' is the overall rate constant and k_i is the rate constant for each barrier. This leads to the now familiar expression

$$P_{AB} = \frac{P_A P_B}{P_A + P_B} \tag{6}$$

for a two-humped barrier, where P_{AB} is the total penetrability, and P_A and P_B are the penetrabilities for each of the two barriers.

For excitations near the top of the barrier, the configuration in the second well may well be represented as an independent compound nucleus with various decay modes, such that Eqs. 5 and 6 are valid. For fully open channels, we see that the Bohr-Wheeler estimate is modified to read

$$\langle \Gamma_f \rangle = \frac{nD}{4\pi} \tag{4'}$$

If we calculate this quantity, using the recommended spacings for spin 3 and spin 4, we find $\langle \Gamma_f \rangle_3$ = 0.092 eV and $\langle \Gamma_f \rangle_4$ = 0.071 eV for each open channel. We can thus infer that, for spin 3, the observed fission width is consistent with two fully open channels, or more than two, if they are only partially open. The observed fission width $\langle \Gamma_f \rangle_4$ corresponds to no more than one fully open channel. This is reasonably consistent with the distribution of widths for the resolved resonances: Keyworth et al [8] reported ν = 2.04 ± 0.65 and 1.27 ± 0.33 for spin-3 and spin-4 fission widths, respectively, based on a fit to the chi-squared distribution with ν degrees of freedom, using the method of maximum likelihood. The Bohr-Wheeler estimate is in surprisingly good agreement with our recommended values of $\langle \Gamma_f \rangle_3$ = 0.184 eV and $\langle \Gamma_f \rangle_4$ = 0.087 eV; we calculate $\langle \Gamma_f \rangle_3$ = 2.04 * 0.092 = 0.188 eV, and $\langle \Gamma_f \rangle_4$ = 1.27 * 0.071 = 0.090 eV (with errors of 30%).

The data of Pattenden and Postma [23] can be used to give additional information on the fission channel quantum numbers. Pattenden and Postma measured angular distributions of fission fragments with aligned target nuclei of ^{235}U, reporting their results in terms of A_2, the coefficient of the P_2 term in the Legendre expansion of the angular distribution. The coefficient A_2 is a function of both J and K, the projection of J on the nuclear symmetry axis.

We find that A_2 is significantly correlated with J_{eff} (at the significance level of 10^{-8} as defined in Table I.) A plot of A_2 versus J_{eff} is shown in Fig. 9. We use a linear least-squares fit to these data, shown by the solid line in Fig. 9, to infer the average value of A_2 for pure spin-3 resonances (J_{eff} = 3.0) and for pure spin-4 resonances (J_{eff} = 4.0), obtaining $\langle A_2 \rangle_3$ = -1.22, $\langle A_2 \rangle_4$ = -2.01. For J = 4, we assume that the lowest two channels, K = 1 and K = 2, are open. Knowing the characteristic A_2 for each J, K (shown as the bars on the right hand side of Fig. 9) enables us to calculate the contributions from each channel; we find $\langle \Gamma_f \rangle_{J,K = 4,1}$ = 0.071 eV, $\langle \Gamma_f \rangle_{4,2}$ = 0.016 eV if the total width is 0.087 eV. We infer that the J,K = 4,1 channel is fully open, the J,K = 4,2 channel is only partially open. For J = 3, we have an apparent inconsistency. We expect three possible channels, for K = 0,1,2, and we expect that if the J,K = 4,1 channel is fully open, the J,K = 3,1 channel (which presumably lies at lower excitation) will also be fully open, with an average fission width of 0.092 eV. With these assumptions, we can solve for $\langle \Gamma_f \rangle_{J,K = 3,0}$ and $\langle \Gamma_f \rangle_{J,K = 3,2}$, finding $\langle \Gamma_f \rangle_{3,0}$ = 0.019 eV, $\langle \Gamma_f \rangle_{3,2}$ = 0.073 eV for a total width of 0.184 eV. Within the error on the least squares fit, we could use $\langle \Gamma_f \rangle_{3,1}$ = $\langle \Gamma_f \rangle_{3,2}$ = 0.092 eV and $\langle \Gamma_f \rangle_{3,0}$ = 0.

These results are not new; essentially they confirm those of the earlier polarization measurements of Keyworth et al [7], who arrived at the same conclusion. But they are not what had been expected. For many years, the assumption was made that the channels open in order of ascending K, following the sequence of octupole vibrational band heads observed near the ground state of even-even nuclides. Why the J,K = 3,0 channel seems to be forbidden remains an unanswered question.

THE VARIATION OF $\langle\alpha\rangle$

The most important result of the present study, that the structure in the fission cross section of ^{235}U can be attributed to the double-humped barrier, and, in particular, to the $J = 4^-$ spin state for s-wave neutron-induced fission, leads to a new understanding of the variation of the capture-to-fission ratio, and to the necessity of a revised treatment of the capture cross section and $\langle\alpha\rangle$. While earlier work [1-4] had strongly suggested that the double-humped barrier might be of importance in causing fluctuations in σ_f for ^{235}U, there was no prescription for treating this effect in an evaluation. For ENDF/B. the approved procedure for treating the fluctuations in the unresolved resonance region and for File 3 (the "smooth" cross sections) is as follows: one looks at the fluctuations in the capture and fission cross sections and holding $\langle\Gamma_\gamma\rangle$ fixed one solves for a pointwise variable $\langle\Gamma_f\rangle$ and $\langle\Gamma_n\rangle$ for one or both spin states which describes the fluctuations, in broad-bin averages, to the desired degree of accuracy. The difficulty, at least with previous versions of ENDF/B, is that $\langle\alpha\rangle$ above 3 keV was given with too coarse a bin structure ($\sim$ 1 keV) to describe the intermediate structure; the result was that the capture and fission cross sections tended to show the same structure, and their ratio, $\langle\alpha\rangle$, was more or less featureless.

The present results suggest a completely different treatment. If the structure in fission is due to enhancement of the 4^- resonances related to the double-humped barrier, the capture and fission cross section structure will show a strong negative correlation, and $\langle\alpha\rangle$ will reflect this in showing pronounced fluctuations; it is hardly necessary to add that we should expect a considerable difference in the calculated self-shielding factors and Doppler coefficients.

The purpose of the present section is to show that evidence exists to support the anticorrelation of the fission and capture cross sections of ^{235}U, and, in particular, to show that it is the J,K = 4,2 component which reflects the intermediate structure in ^{235}U fission. We begin by showing, in Fig. 10, the fission and capture cross sections (multiplied by $\sqrt{E}$ for greater clarity) from 0.1 to 1 keV as reported by Gwin et al [24]. The correlation coefficient is strong (-0.494) but hardly conclusive, since there are only nine data points. We also calculated the correlation coefficient between $\langle\alpha\rangle$ from ENDF/B-IV and J_{eff} from 80 eV to 1 keV, finding much the same result: the correlation is strong (-0.511) but not conclusive, because there are too few data points below 1 keV, and the bin structure above 1 keV is too coarse to show the effect.

Next we note, as shown in Fig. 11, the data reported by Pattenden and Postma on the variation of A_2 below 2 keV. The data have very large uncertainties at the highest energies, but they seem to suggest a trend, a lowering of $-A_2$ with increasing energy. If we calculate the expected variation of A_2 using the double-hump barrier parameters of Back et al [25] for the compound nucleus ^{236}U, we find that there is no way we can get a variation much larger than 1% in 2 keV, except by assuming second-well enhancement.

If we make the assumption that any variation in A_2 is due to the spin-4 component, A_2 for spin 3 remaining fixed at -1.22, then we can solve for $\langle A_2 \rangle_4$ as a function of energy. This is shown in Fig. 12 over the energy region 0.1 - 1.5 keV; plotted in the same figure is $\langle \alpha \rangle$ reported by Gwin and $\langle \alpha \rangle$ given in ENDF/B-IV. The positive correlation is obvious: $\langle \alpha \rangle$ is low when the J,K = 4,2 channel is large (low values of $-A_2$); again the correlation is not conclusive because there are too few data points. No one piece of evidence is conclusive, yet they all point in the same direction: the fluctuations in σ_f are due to second-well enhancement of the J,K = 4,2 channel, which is reflected in $\langle \alpha \rangle$.

RECOMMENDATIONS FOR ENDF/B-V

To use the present results in the evaluation of the unresolved resonance region requires a change in the approved procedure, and, unfortunately, in the processing codes which use ENDF/B. The problem is that width-fluctuation corrections are not properly made if the two spin-4 fission channels have different widths. A change in procedure is not possible for ENDF/B-V because of deadlines which the evaluators must meet, but we shall outline what we consider to be deficiencies of the present treatment for consideration in the future. The present format allows a pointwise variable (in energy) average neutron width with one or two degrees of freedom, to account for structure in the total and elastic scattering cross sections, a fixed $\langle \Gamma_\gamma \rangle$ with an infinite number of degrees of freedom, and a pointwise variable average fission width with an integral number of degrees of freedom for each spin state, to account for structure in $\langle \sigma_f \rangle$ and $\langle \alpha \rangle$. To generate the average fission, capture, and elastic scattering cross sections from relatively coarse binned data which reflect the structure, one uses the code UR [26], which performs the integrals over the appropriate chi-squared distributions to obtain width-fluctuation corrections, and then uses an iterative procedure to extract the appropriate average widths which fit the binned data. The most time-consuming part of the code is the width-fluctuation calculation. If one performed this calculation from first principles; it would involve a multiple integral over a Porter-Thomas distribution for each of the partial widths which may exist. The code UR contains an expression by Dresner [27], which uses the superposition theorem for chi-squared distributions to reduce the multiple integral to a single integral, with the restriction that the number of degrees of freedom be integral. We had hoped, by a suitable definition of a non-integral number of degrees of freedom to describe the case $\langle \Gamma_f \rangle_{J,K\,=\,4,1} \neq \langle \Gamma_f \rangle_{J,K\,=\,4,2}$, that the Dresner expression could still be used, but unfortunately it does not give the right answer for the width fluctuation correction integrals unless $\langle \Gamma_f \rangle_{4,1} = \langle \Gamma_f \rangle_{4,2}$ or unless one of the two partial widths is zero. We find that the width-fluctuation integrals given by the Dresner expression differ from the correct integrals by as much as 5% for ν_{eff} non-integral. Perhaps there is a definition of ν_{eff} which would allow general use of the Dresner formula, but we have not found it.

We recommend that, after ENDF/B-V, use of the Dresner expression be discontinued, both in UR and in the processing codes which use ENDF/B, in favor of a somewhat more complicated but presumably more accurate representation by Shaker and Lukyanov [29], which treats the case that the reaction channels can be divided into a small number of groups with a different average width for each of the groups. Alternatively, one might consider an approach similar to the quick and simple one we devised for testing the Dresner formula: we actually carried out the triple integration, replacing each integral by a weighted sum over 20 levels judiciously chosen from the appropriate chi-squared distribution. We found that we could calculate width-fluctuation corrections in agreement with the Dresner formula (where it is applicable) to less than 1% in all cases we tried, and generally the agreement extended to the fourth decimal place. Furthermore, most of the computer time was spent in evaluating the Dresner formula. Additional time savings might be achieved by selecting the twenty widths from a non-integral chi-squared function, in which case one reduces the triple sum to a double sum.

If the problem of calculating width-fluctuation corrections for a non-integral number of fission channels were solved, then the s-wave parameterization given in Table II could be used as a starting point for the extraction of energy dependent widths in the unresolved region.

Table II also contains recommended p-wave parameters. To obtain these, we chose p-wave strength functions consistent with an extrapolation of the $p^{1/2}$ and $p^{3/2}$ optical model parameters of Lagrange [30] to ^{238}U, a constant radiation width, equal to that for s-waves, and fission widths which give a reasonable representation of $\langle\alpha\rangle$ above the unresolved resonance region. The results of a calculation based on this parameterization are shown in Fig. 13. Again, it should be pointed out that these are initial guesses only, and are open to modification as required by the detailed fitting of the structure. It is interesting to note that the recommendations made by Pitterle et al [21] for ENDF/B-III are remarkably close to those shown in Table II, especially considering that essentially none of the data we have used were available to them at that time. It also might be noted that we deliberately refrained from studying Pitterle's report until the present study was completed.

For ENDF/B-V, we are still restricted to integral values of the number of fission degrees of freedom because of the widespread use of the Dresner formula in treating width-fluctuation corrections. We recommend that both $\langle\Gamma_f\rangle_{J^\pi,K} = 4^-,1$ and $\langle\Gamma_f\rangle_{J^\pi,K} = 4^-,2$ be varied together, with $\nu = 2$. This should be a much better representation than earlier versions which varied $\langle\Gamma_f\rangle$ for both spins, and, while it is not strictly accurate, may be a reasonable compromise.

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REFERENCES

1. M. G. CAO, E. MIGNECO and J. P. THEOBOLD, *Phys. Letters*, 27B, 409 (1968).

2. R. B. PEREZ, G. de SAUSSURE, E. G. SILVER, R. W. INGLE and H. WEAVER, *Nuc. Sci. Eng.*, 55, 203 (1974).

3. G. D. JAMES, G. de SAUSSURE AND R. B. PEREZ, *Trans. Am. Nuc. Soc.*, 17, 495 (1973); see also *UKNDC* (74) P63, 38 (1974).

4. E. MIGNECO, P. BONSIGNORE, G. LANZANÒ, J. A. WARTENA and H. WEIGMANN, *Nuclear Cross Sections and Technology*, NBS Special Publication 425, 607 (1975).

5. G. A. KEYWORTH, J. R. LEMLEY, C. E. OLSEN, F. T. SEIBEL, J. W. T. DABBS and N. W. HILL, *Phys. Rev.*, C8, 2362 (1973).

6. G. A. KEYWORTH, J. R. LEMLEY, C. E. OLSEN, F. T. SEIBEL, J. W. T. DABBS and N. W. HILL, *Physics and Chemistry of Fission 1973*, I, 97, IAEA, Vienna (1974).

7. G. A. KEYWORTH, C. E. OLSEN, F. T. SEIBEL, J. W. T. DABBS and N. W. HILL, *Phys. Rev. Letters*, 31, 1077 (1973).

8. G. A. KEYWORTH, C. E. OLSEN, J. D. MOSES, J. W. T. DABBS and N. W. HILL, *Nuclear Cross Sections and Technology*, NBS Special Publication 425, 576 (1975).

9. F. J. DYSON and M. L. MEHTA, *J. Math. Phys.*, 4, 703 (1963).

10. A. P. JAIN and J. BLONS, *Nuc. Phys.*, A242, 45 (1975).

11. S. MUGHABGHAB and D. I. GARBER, BNL 325, Third Ed., Vol. 1 (1973).

12. J. R. SMITH and R. C. YOUNG, ANCR-1044 (1971).

13. J. T. REYNOLDS, KAPL-M-7396 (1975), and Private Communication.

14. J. D. GARRISON, *Phys. Rev. Letters*, 29, 1185 (1972).

15. J. FELVINCI, E. MELKONIAN and W. W. HAVENS, JR., *Nuclear Cross Sections and Technology*, NBS Special Publication 425, 580 (1975).

16. R. L. BRAMBLETT and J. B. CZIRR, *Nuc. Sci. Eng.*, 35, 350 (1969).

17. D. R. KOENIG and L. L. CARTER, *Trans. Am. Nuc. Soc.*, 17, 491 (1973).

18. D. E. CULLEN and E. F. PLECHATY, *Trans. Am. Nuc. Soc.*, 17, 490 (1973).

19. M. S. MOORE, *Nuclear Cross Sections and Technology*, NBS Special Publication 425, 129 (1975).

20. M. S. MOORE and O. D. SIMPSON, *Neutron Cross Section Technology*, CONF-660303, 840 (1966).

21. T. A. PITTERLE, N. C. PAIK and C. DURSTON, WARD-4210T4-1 (1971).

22. H. EYRING, *Rev. Mod. Phys.*, 34, 616 (1962).

23. N. J. PATTENDEN and H. POSTMA, *Nuc. Phys.*, A167, 225 (1971).

24. R. GWIN, E. G. SILVER, R. W. INGLE and H. WEAVER, *Nuc. Sci. Eng.*, 59, 79 (1976).

25. B. B. BACK, O. HANSEN, H. C. BRITT and J. D. GARRETT, *Phys. Rev.*, C9, 1924 (1974).

26. E. M. PENNINGTON, Private Communication to M. Bhat, March (1973).

27. L. DRESNER, ORNL-2659 (1959).

28. L. WILETS, *Physics Rev. Letters*, 9, 430 (1962).

29. M. O. SHAKER and A. A. LUKYANOV, *Physics Letters*, 19, 197 (1965).

30. CH. LAGRANGE, in "Critique of Nuclear Models and Their Validity in the Evaluation of Nuclear Data," *EANDC Topical Discussion*, Tokyo, March (1974).

TABLE I

Correlation coefficients and significance levels for the correlation of spin-3 and spin-4 data with structure in ^{235}U σ_f, from 8 - 25 keV. In this table, the significance level is the probability that the observed correlation or larger would occur with a randomly selected sample.

Energy Range (keV)	Bin Width (keV)	$\rho(N_3,\Sigma)$	Significance of $\rho(N_3,\Sigma)$	$\rho(N_4,\Sigma)$	Significance of $\rho(N_4,\Sigma)$
8.0 - 10.4	0.12	-0.01617	$\sim$0.50	0.7048	0.0003
10.4 - 12.8	0.12	0.2148	0.18	0.6148	0.002
12.8 - 15.2	0.12	0.0889	0.35	0.3815	0.05
15.2 - 20.0	0.24	0.1996	0.20	0.7111	0.0002
20.0 - 24.8	0.24	0.2336	0.16	0.7443	0.0001
24.8 - 34.4	0.48	0.2864	0.11	0.8194	<0.00001

TABLE II

Unresolved Resonance Parameters for ^{235}U

S_o	$= \langle \Gamma n^\circ/D \rangle \sim 1.0 \times 10^{-4}$ and variable, depending on structure in $\langle \sigma_T \rangle$.
$S_{1,1/2}$	$= \langle \Gamma n^1/D \rangle_{1/2} = 1.26 \times 10^{-4}$
$S_{1,3/2}$	$= \langle \Gamma n^1/D \rangle_{3/2} = 1.76 \times 10^{-4}$
r_o	= 9.5663 fm (unchanged from ENDF/B-IV)
$D_{J=2}$	= 1.6135 eV
$D_{J=3}$	= 1.1525 eV
$D_{J=4}$	= 0.8958 eV
$D_{J=5}$	= 0.7334 eV
$\langle \Gamma_f \rangle_{3^-}$	= 0.184 eV, $\nu = 2$
$\langle \Gamma_f \rangle_{J^\pi,K = 4^-,1}$	= 0.071 eV, $\nu = 1$
$\langle \Gamma_f \rangle_{J^\pi,K = 4^-,2}$	$\sim$ 0.04 eV and variable, depending on structure in $\langle \sigma_f \rangle$ and $\langle \alpha \rangle$, $\nu = 1$
$\langle \Gamma_\gamma \rangle$	= 0.035 eV*, $\nu = \infty$ (unchanged from ENDF/B-IV)
$\langle \Gamma_f \rangle_{2^+}$	= 0.513 eV, $\nu = 4$
$\langle \Gamma_f \rangle_{3^+}$	= 0.276 eV, $\nu = 3$
$\langle \Gamma_f \rangle_{4^+}$	= 0.285 eV, $\nu = 4$
$\langle \Gamma_f \rangle_{5^+}$	= 0.173 eV, $\nu = 3$

*Calculations shown in Fig. 13 used $\langle \Gamma_\gamma \rangle$ = 0.037 eV.

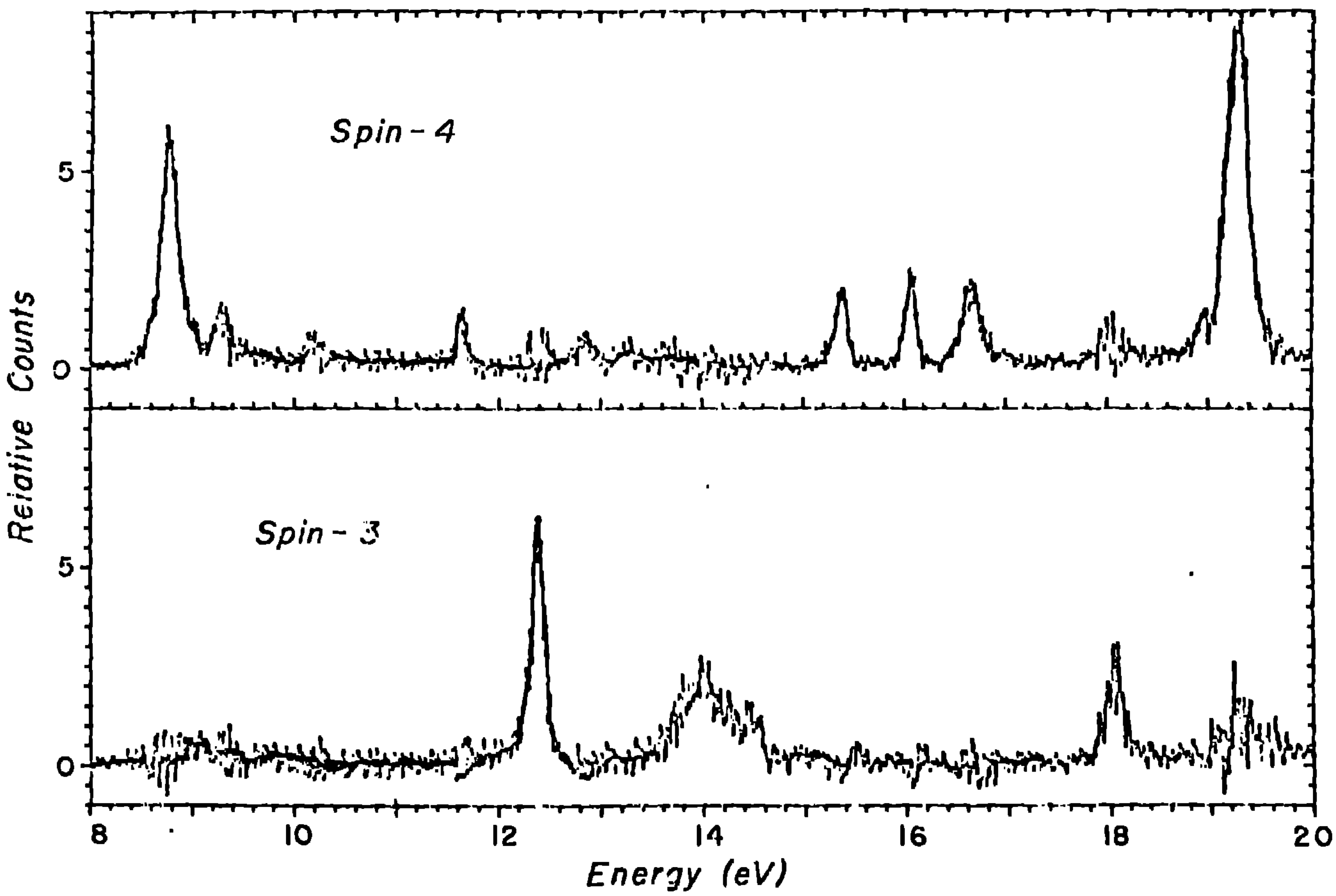

Fig. 1. Spin-separated resonance structure in the fission of ($^{235}U + n$) versus neutron energy in the energy range from 8 - 20 eV.

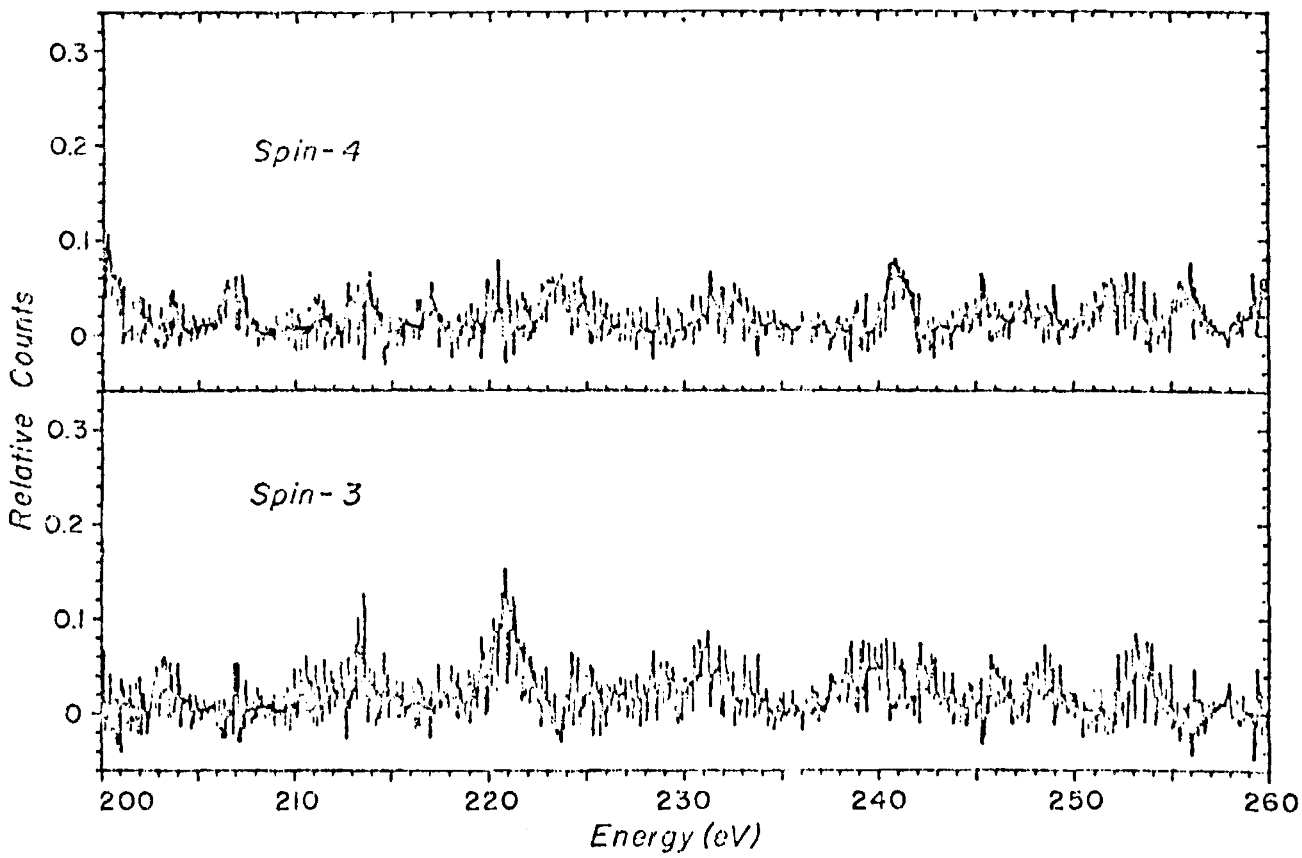

Fig. 2. Spin-separated resonance structure in the fission of ($^{235}U + n$) versus neutron energy in the energy range from 200 – 260 eV.

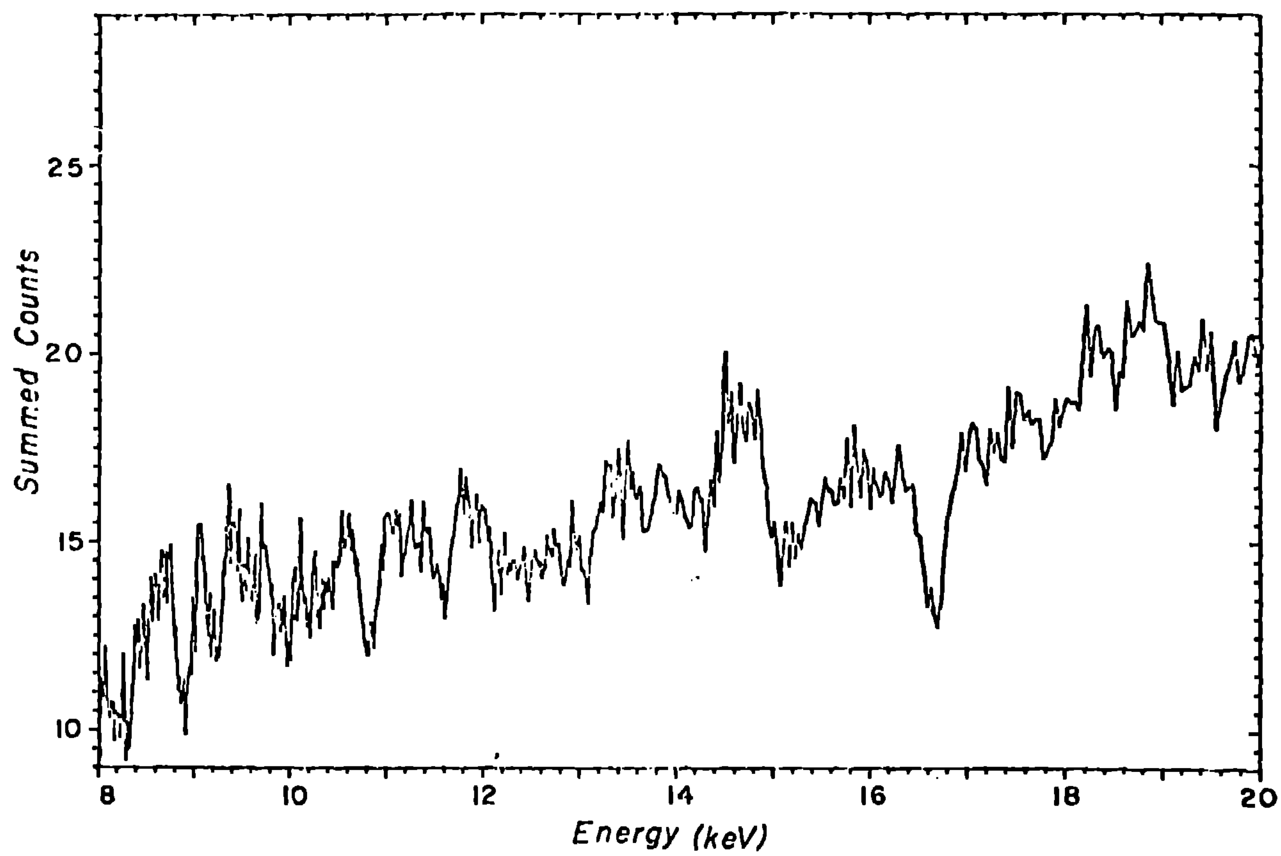

Fig. 3. Summed counts (spin-3 enhanced plus spin-4 enhanced count rates) observed in the fission of ($^{235}U + n$) versus neutron energy in the energy range from 8 - 20 eV. The structure corresponds to the well known fluctuations previously observed.[1-4]

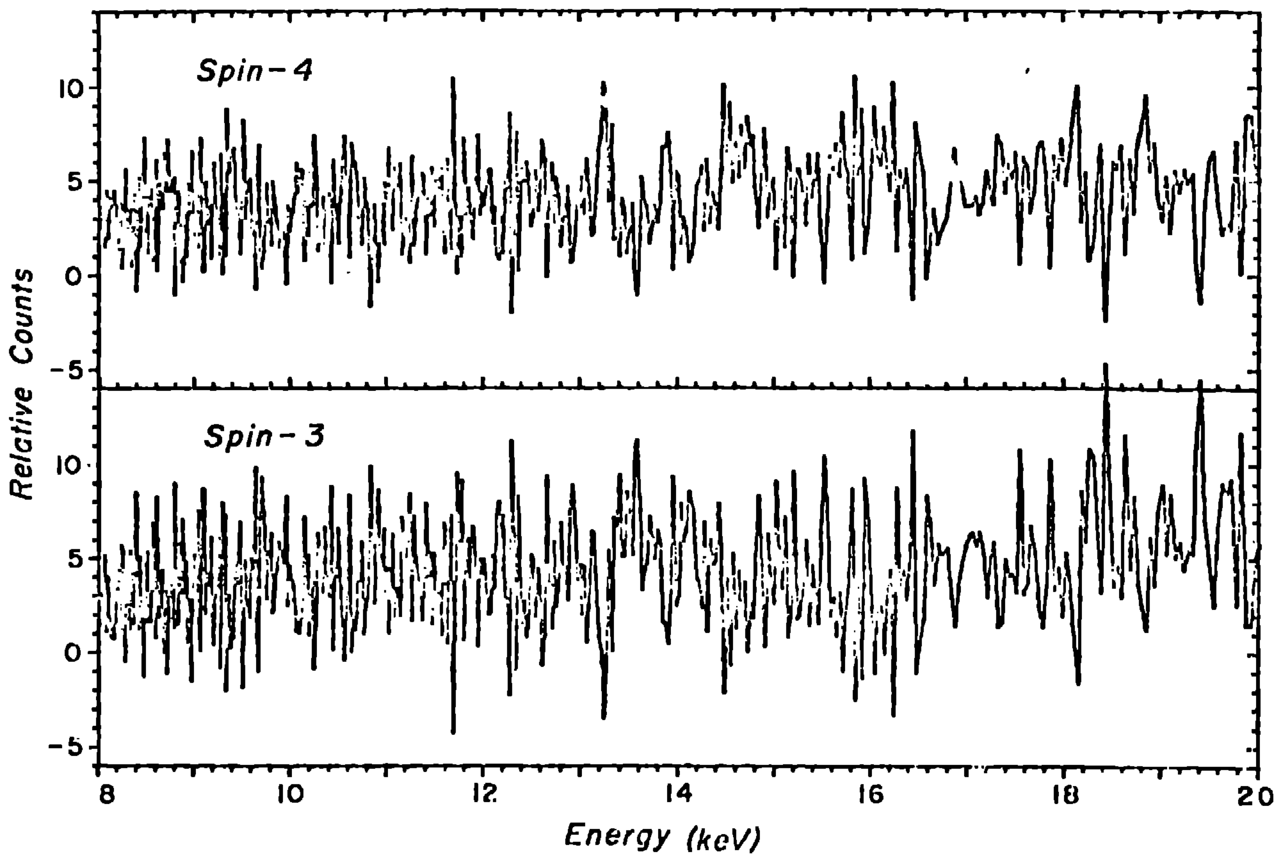

Fig. 4. Spin-separated count rates in the fission of (^{235}U + n) versus neutron energy in the energy range from 8 – 20 keV. Except for the cluster between 14 and 15 keV, which is clearly spin 4, it is not obvious that either of these curves correlates with that shown in Fig. 3.

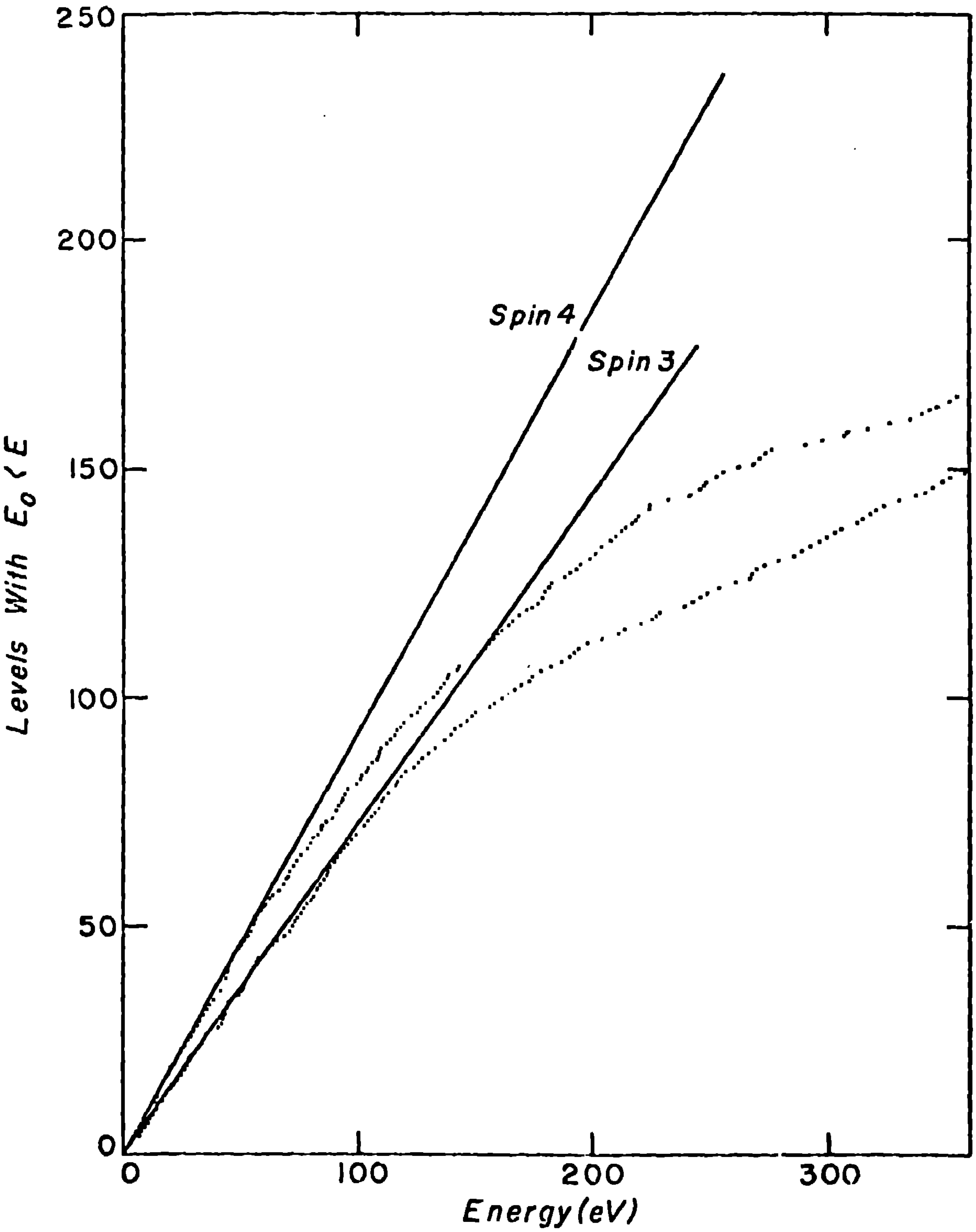

Fig. 5. Observed resonance spacing distribution in ($^{235}U + n$) below 360 eV. Data points give the number of levels having a resonance energy less than the energy shown on the abscissa, and correspond to the tips of the stairs in the usual stairstep plot. The solid lines represent a fit to the data points below 60 eV, and show the expected (2J + 1) slope.

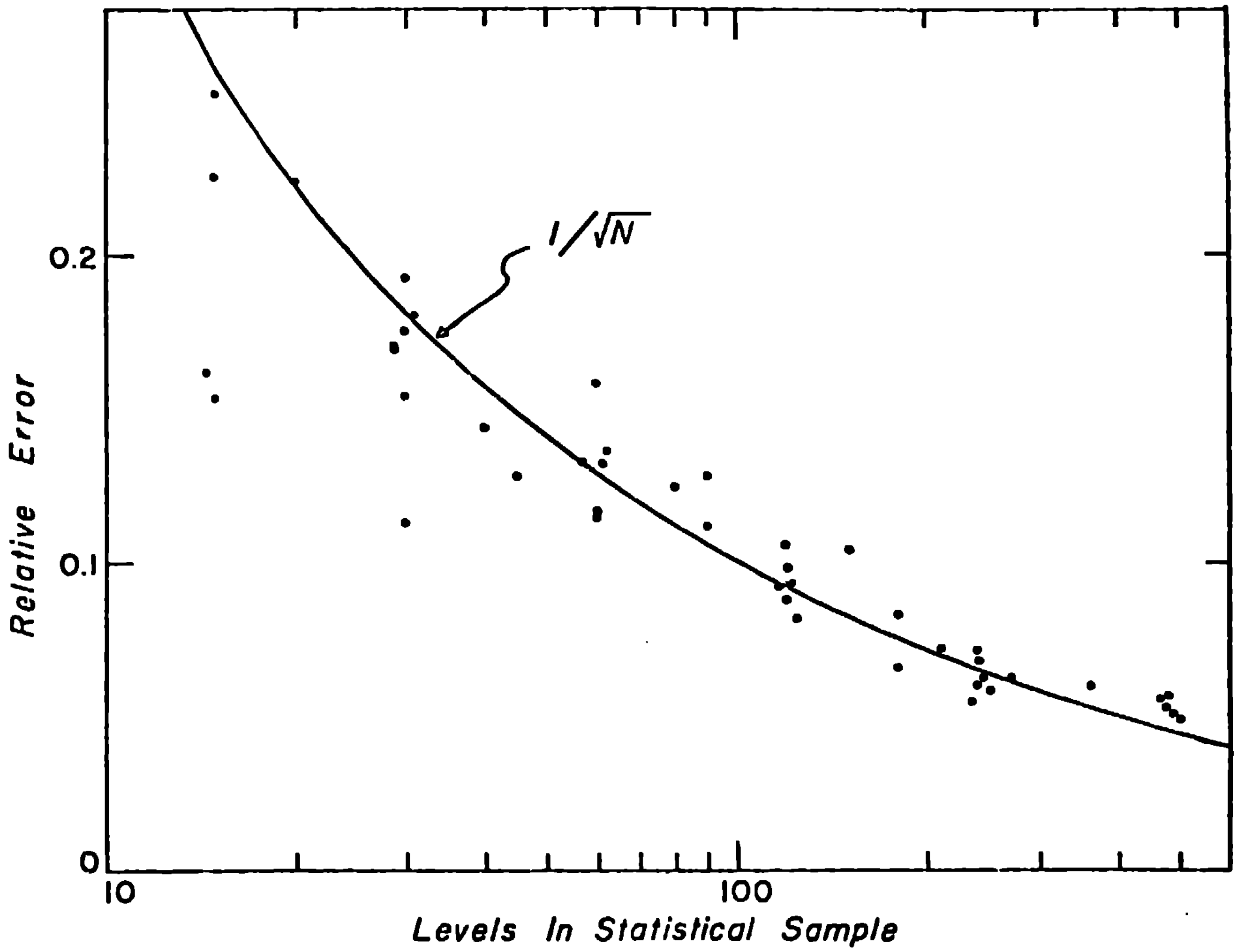

Fig. 6. Distribution of relative error in a random sampling of levels from a Porter-Thomas distribution for the missing-level estimator described in the text, versus the number of levels in the statistical sample. The solid line shows the curve $\Delta N/N = 1/\sqrt{N}$, where N is the number of levels in the sample.

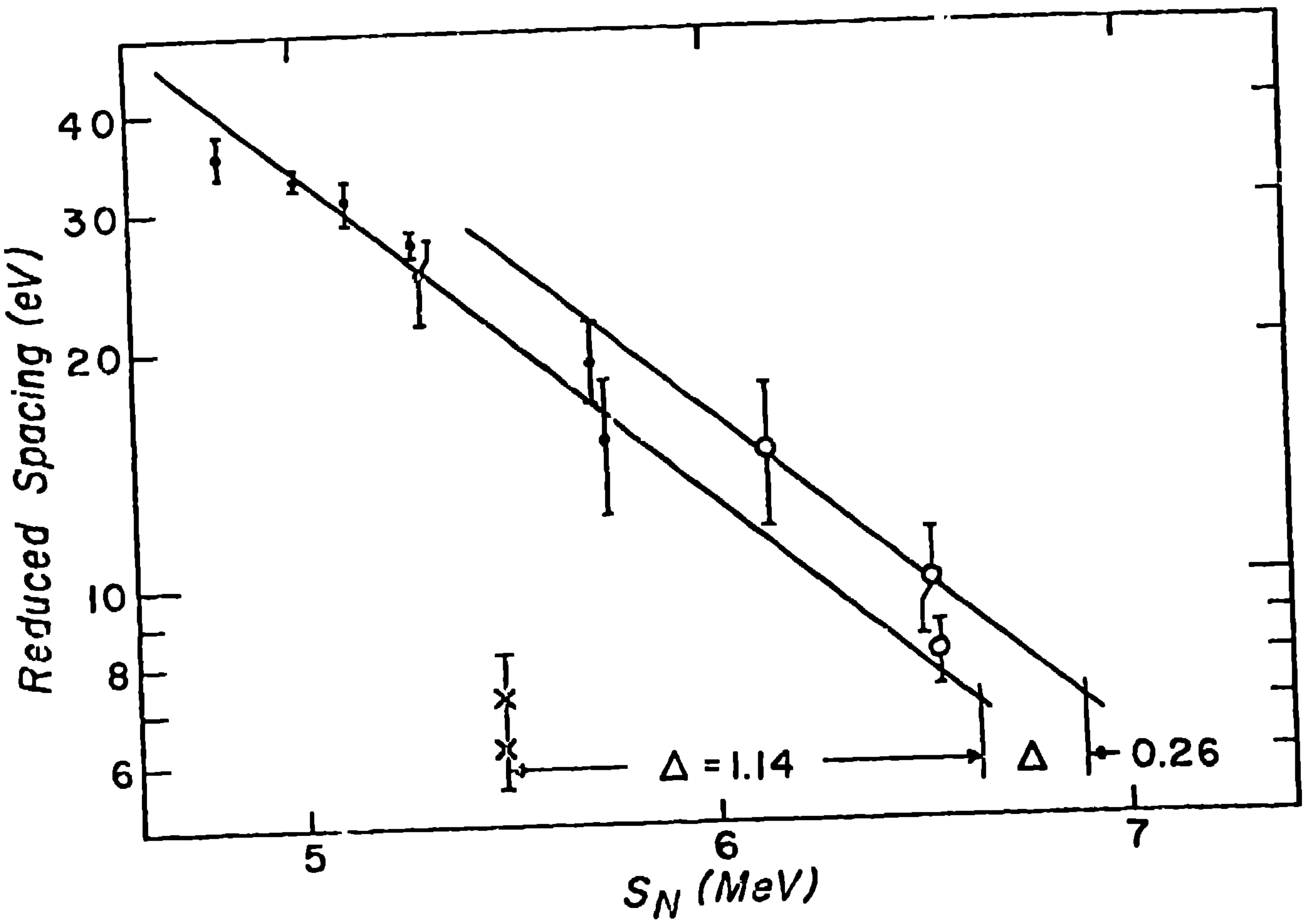

Fig. 7. Reduced level spacing D(2J + 1) for several actinide nuclides for which D is reasonably well determined, versus the neutron separation energy of the compound system. Open circles represent even-even, solid circles even-odd, and x's odd-odd compound nuclei. The deltas represent the energy shift which is necessary to make the points follow the same curve.

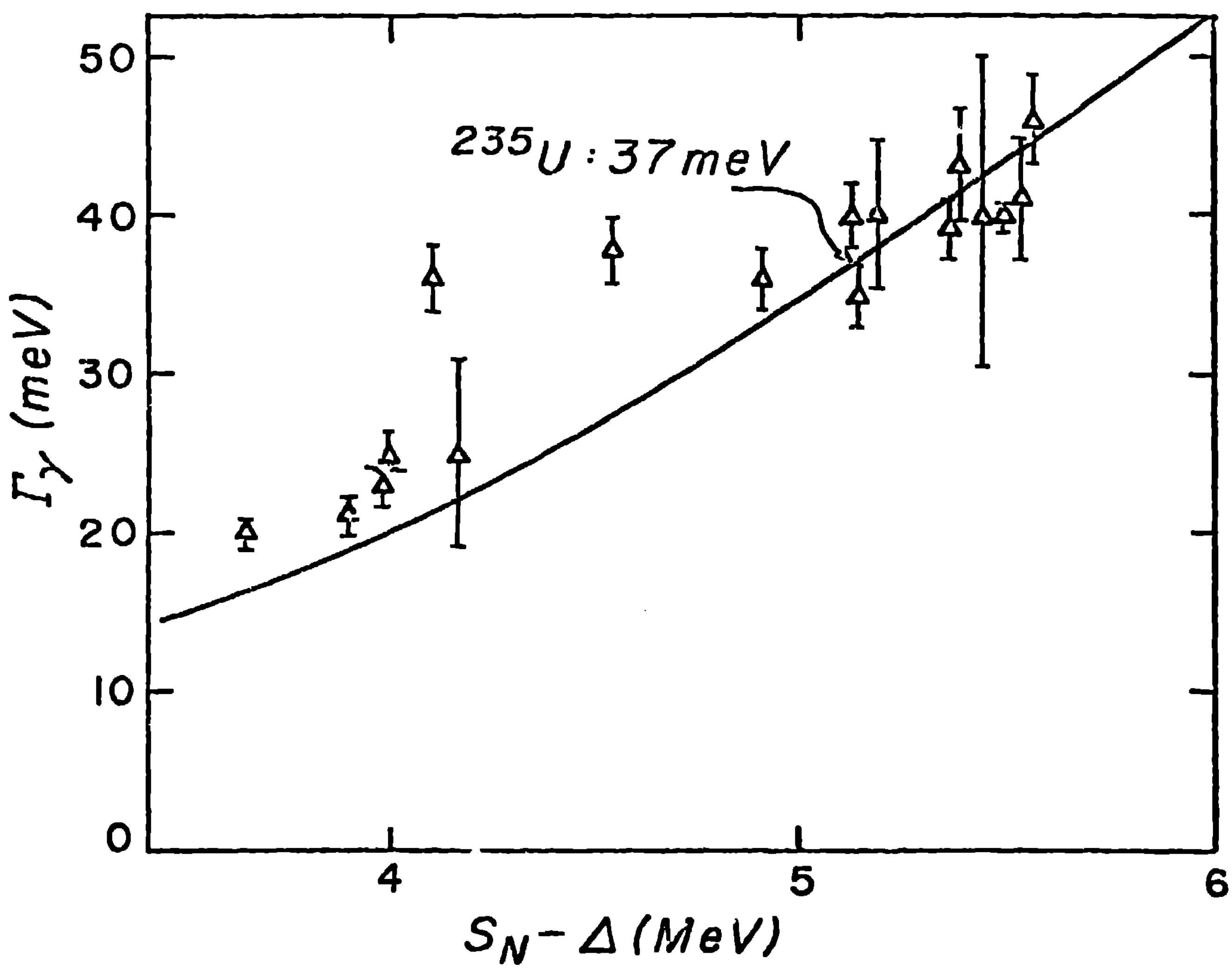

Fig. 8. The variation of the radiative capture width for selected resonances of isotopes of Th, Pa, U, Np, Pu, and Am, versus the neutron separation energy of the compound system less the correction term determined from Fig. 7. The solid line shows the results of a model calculation of the radiative capture width versus excitation energy, normalized to these data.

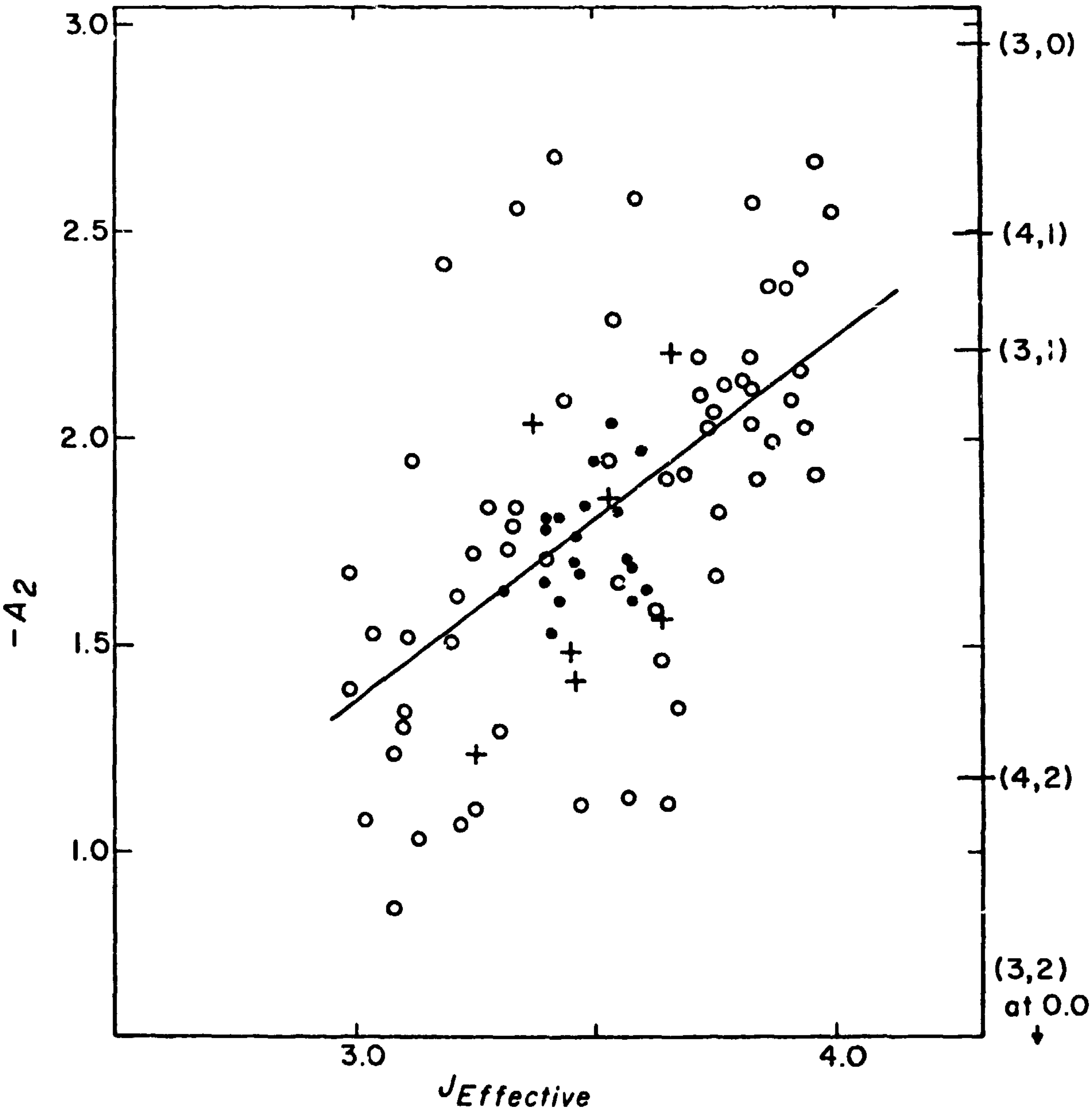

Fig. 9. The variation of A_2 from Pattenden and Postma versus $J_{effective} = 3 + \sigma_4/(\sigma_3 + \sigma_4)$. The straight line shows a linear least-squares fit to these data. The open circles show A_2 data for resonance structure, the closed circles data for the unresolved region below 2 keV, and the plus signs data for the between-resonance background regions reported by Pattenden and Postma.

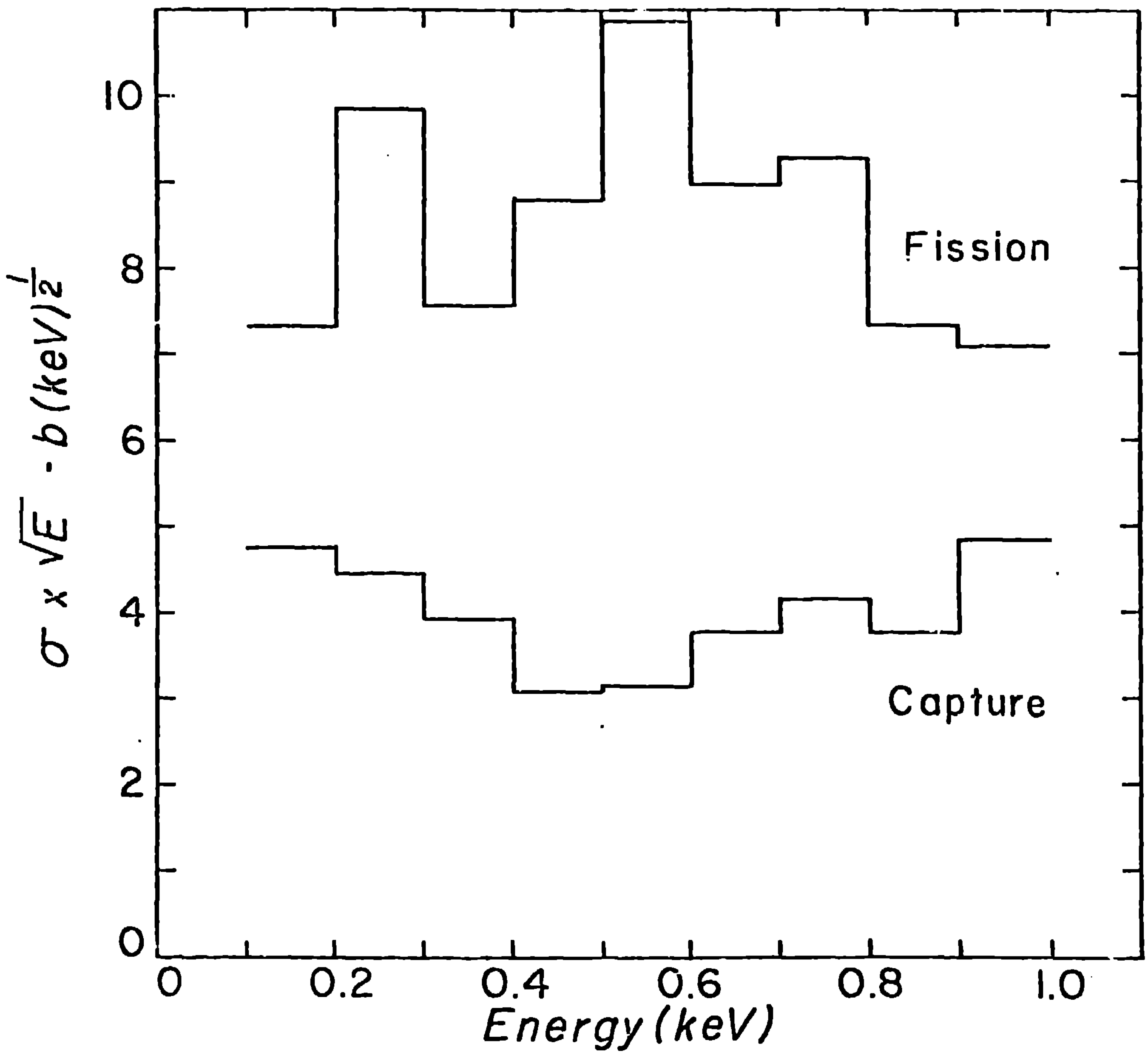

Fig. 10. Fission and capture cross sections of ^{235}U between 0.1 and 1 keV, multiplied by the square root of the neutron energy.

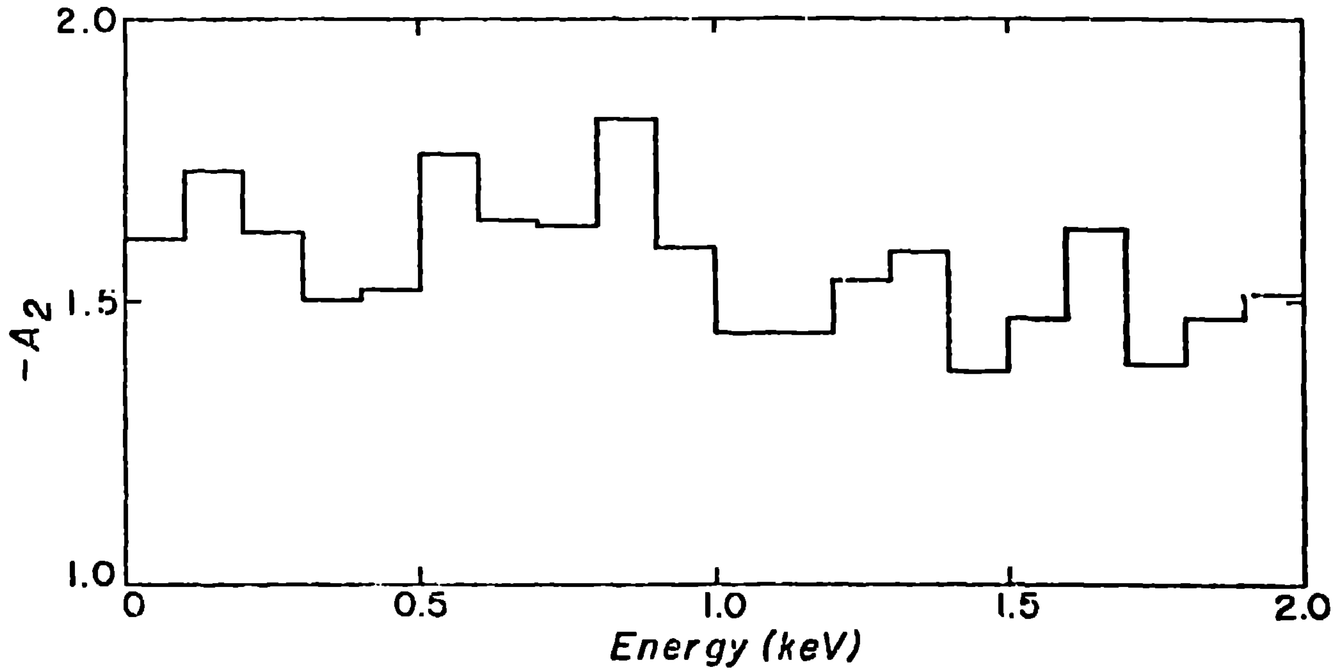

Fig. 11. The variation of A_2, from Pattenden and Postma, versus neutron energy below 2 keV.

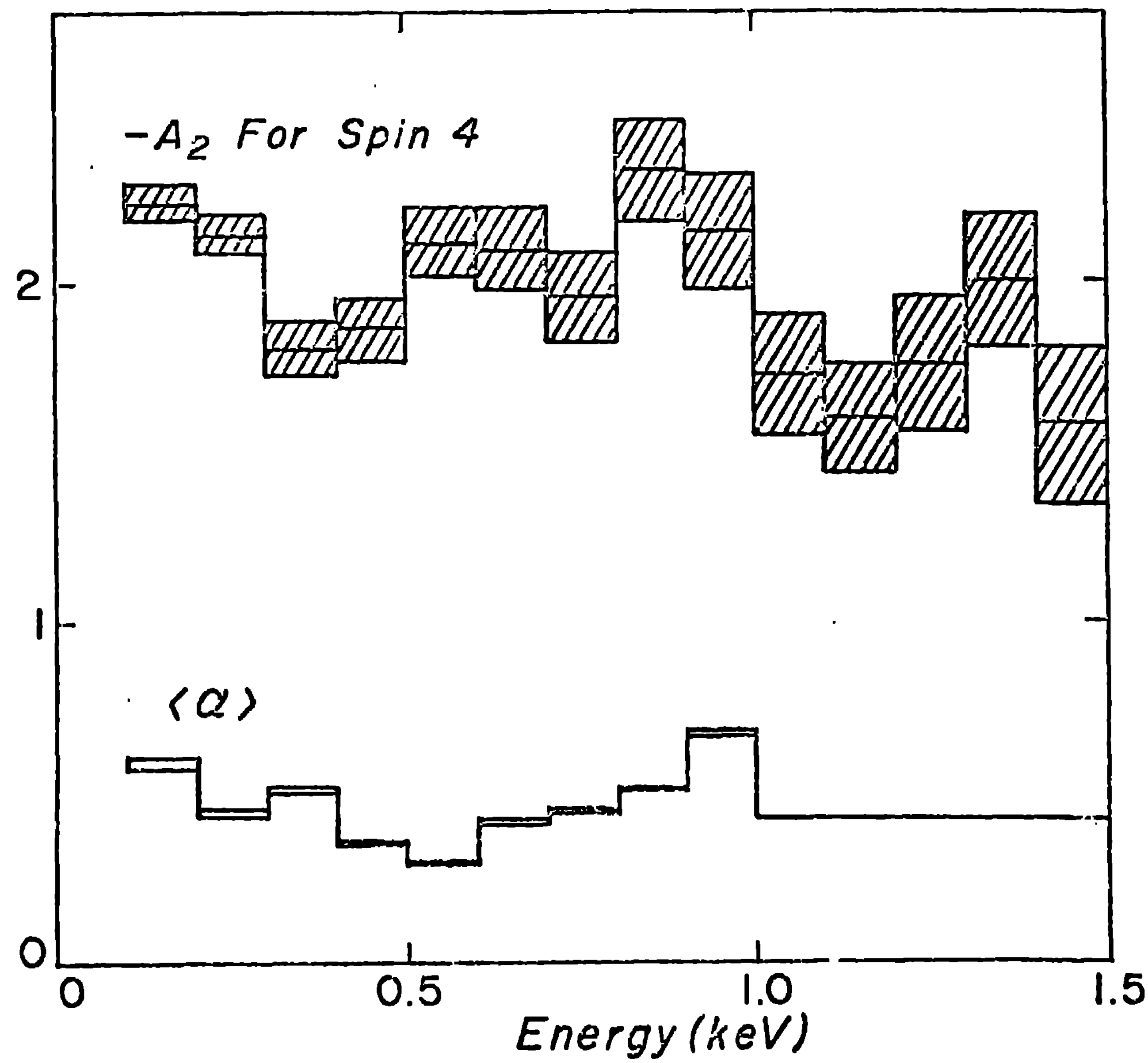

Fig. 12. The variation of A_2 from Pattenden and Postma, versus neutron energy below 1.5 keV, calculated under the assumption that the variation is entirely attributable to the spin-4 component. Plotted as the lower curve in this figure is $\langle\alpha\rangle$, the capture-to-fission ratio.

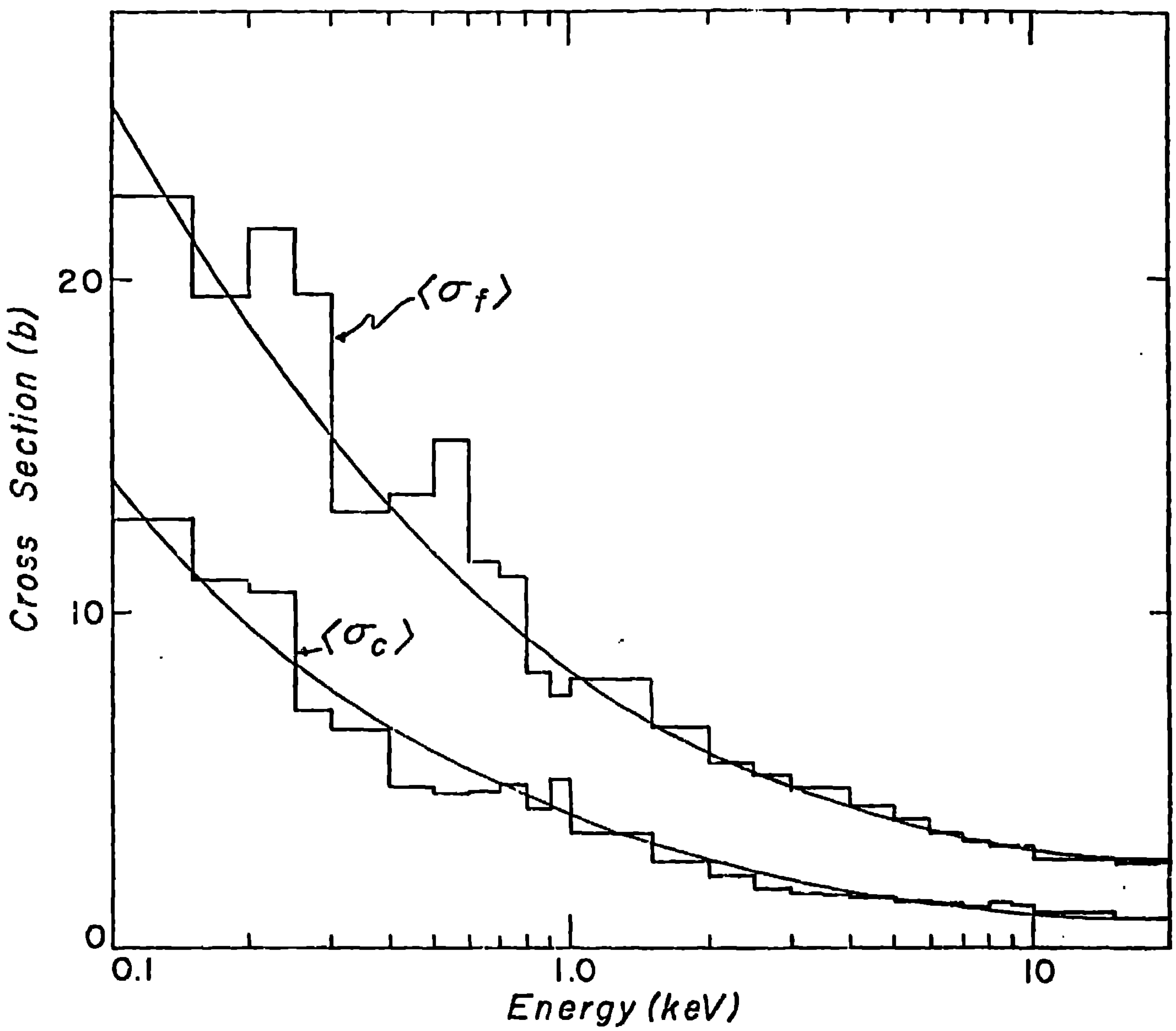

Fig. 13. The fission and capture cross sections of ^{235}U below 25 keV. The histograms reflect the cross section structure reported in ENDF/B-IV; the smooth curves are calculated from parameters in Table II.